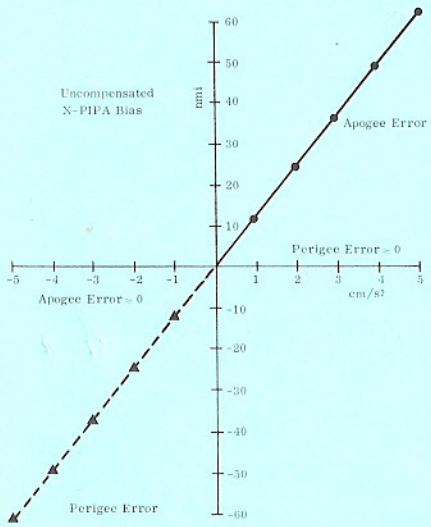
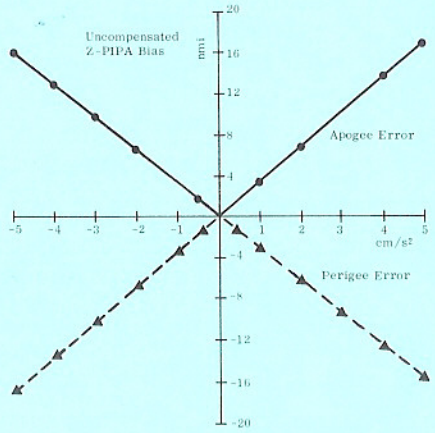
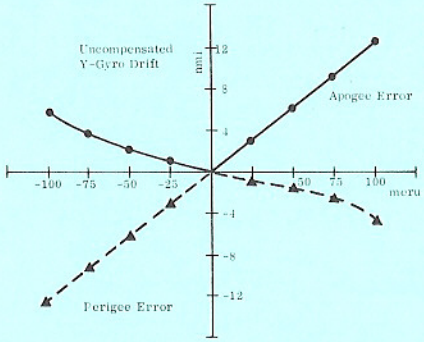
PERTURBATIONS

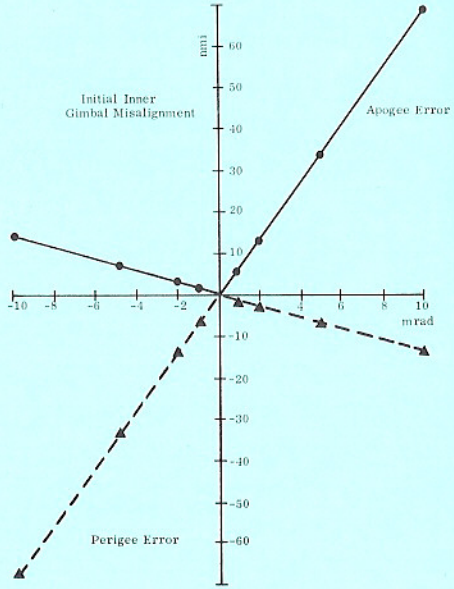
BOOST MONITOR
TLI
LUNAR ORBIT
LM DESCENT
LM ASCENT
TEI
ENTRY

BOOST MONITOR ERRORS
(Significant IMU Errors Only)





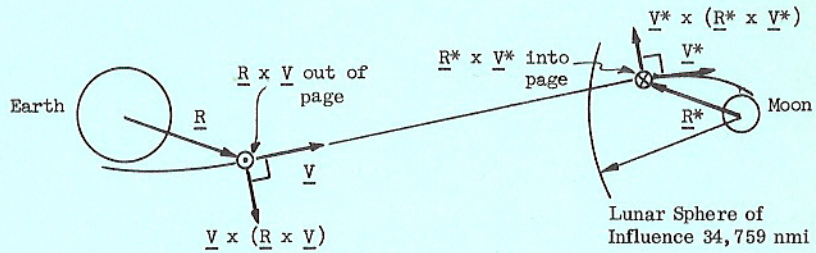




ERRORS IN PERICYNTHION
DUE TO VELOCITY ERRORS DURING TRANSLUNAR COAST

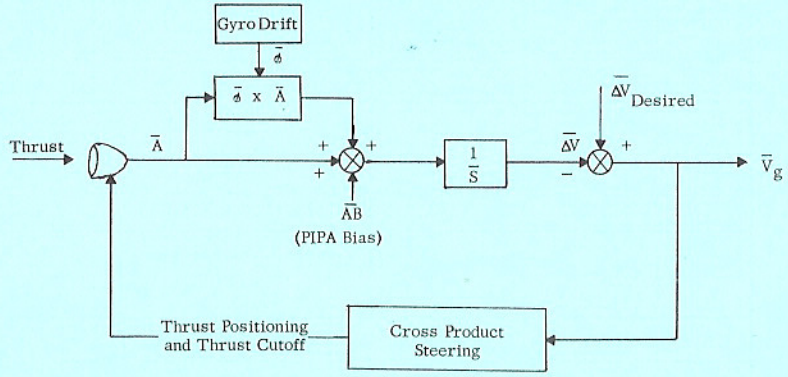
DIRECTION OF VELOCITY ERROR	ERROR IN PERICYNTHION (nmi/ft/s)
ERROR EXISTING AT TLI CUTOFF (2:38:50.0 GET)	
Along $\underline{V}$	129.35
Along $\underline{\bar{R}} \times \underline{V}$	0.38
Along $\underline{\bar{V}} \times (\underline{R} \times \underline{V})$	-4.10
ERROR EXISTING AT MCC-1 (TLI + 9 hours) (11:38:50 GET)	
Along $\underline{V}$	33.63
Along $\underline{\bar{R}} \times \underline{V}$	5.48
Along $\underline{\bar{V}} \times (\underline{R} \times \underline{V})$	-9.38
ERROR EXISTING AT MCC-2 (TLI + 28 hours) (30:38:50 GET)	
Along $\underline{V}$	19.03
Along $\underline{\bar{R}} \times \underline{V}$	4.74
Along $\underline{\bar{V}} \times (\underline{R} \times \underline{V})$	-6.99
ERROR EXISTING AT MCC-3 (LOI - 22 hours) (52:28:38 GET)	
Along $\underline{V}$	9.70
Along $\underline{\bar{R}} \times \underline{V}$	2.73
Along $\underline{\bar{V}} \times (\underline{R} \times \underline{V})$	-3.33
ERROR EXISTING AT MCC-4 (LOI - 5 hours) (69:28:38 GET)	
Along $\underline{V}^*$	0.36
Along $\underline{\bar{R}}^* \times \underline{V}^*$	0
Along $\underline{\bar{V}}^* \times (\underline{R}^* \times \underline{V}^*)$	2.86

* $\underline{R}$ and $\underline{V}$ are with respect to moon centered inertial system.



EFFECT OF INSTRUMENT ERRORS ON THRUST GUIDANCE

The thrust guidance used on Apollo, in simplified form, is as follows



The above steering loop attempts to drive V_g (velocity to be gained) to zero. The inability of the steering loop to drive V_g to zero will result in velocity residuals at the end of the burn as displayed in P-40.

Since the thrust guidance loop performs its calculations on the difference between ΔV measured and ΔV desired, the velocity residuals displayed by P-40 at the end of each burn are not caused by instrument errors but by the inability of the steering loop to drive V_g to zero.

Instrument errors result in a difference between the measured incremental velocity change and the true incremental velocity change. Assuming perfect cross product steering ($V_{Residuals}$ = 0 in P-40) the resultant orbit will differ from the planned orbit. A measure of the difference between the actual and planned orbits is the error between actual and planned apolune and perilune.

The following tables relate incremental spacecraft velocity errors, during lunar orbit maneuvers, to errors in apolune and perilune. Knowing apolune and perilune errors, the table is used to obtain the required error in velocity, expressed in body axes coordinates. This velocity error is then converted to possible instrument errors by using the table relating PIPA bias and gyro drift to spacecraft velocity.

For Example: Assume a perilune error of +5.0 n mi and zero spacecraft residuals, as shown in P-40, after the DOI maneuver.

1. From the "Lunar Orbit Maneuver Perturbation" table, the most sensitive spacecraft velocity error is ΔV_x.

2. The required velocity error is

$$\Delta V_x = \frac{5.0 \text{ n mi}}{-.70 \text{ n mi/ft/sec}} = -7.14 \text{ ft/sec.}$$

3. The weighting factors relating instrument errors to velocity errors yield

$$-0.69 \times 10^{-2} AB_x \times 53.2 - 3.21 \times 10^{-2} AB_z \times 53.2 = -7.14 \text{ ft/sec}$$

$$-.37 AB_x - 1.71 AB_z = -7.14 \text{ ft/sec}$$

Therefore:

$$\text{Uncompensated } AB_x = +19 \text{ cm/sec}^2$$

or

$$\text{Uncompensated } AB_z = +4.17 \text{ cm/sec}^2$$

EFFECT OF INSTRUMENT ERRORS ON THRUST GUIDANCE (CONTINUED)

The "Lunar Orbit Maneuver Perturbation" table also gives the transformation from stable member space to spacecraft body control axes space as follows:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{S/C\ CSM} = \left[R_X\ (-7.25°)\right] \left[SM\text{-}NB\right] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{SM}$$

where:

$\left[SM\text{-}NB\right]$ = Transformation from stable member to the nav base using gimbal angles

$\left[R_X\ (-7.25°)\right]$ = Rotation matrix about X_{NB} of -7.25 degrees to transform from nav base space to spacecraft control axes space for the CSM.

For the LM

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{S/C\ LM} = \left[SM\text{-}NB\right] \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_{SM}$$

The purpose of this transformation is to indicate the location of the stable member with respect to thrust. The assumed gimbal angles are as follows:

	OUTER GIMBAL (degrees)	INNER GIMBAL (degrees)	MIDDLE GIMBAL (degrees)
LOI	0	0	0
DOI	0	282	359
CIRCULARIZATION	0	141	358
LOPC 1	0	0	0
LM TPI	0	103	0
LOPC 2	0	0	0
SHAPING	3	58	3
TEI	180	0	0

For all maneuvers thrust is assumed along $+X_{S/C}$ axis.

APOLLO 16 LUNAR ORBIT MANEUVER PERTURBATIONS

MANEUVER	BODY AXES VELOCITY ERRORS	ERROR IN PERILUNE (nmi/ft/s)	ERROR IN APOLUNE (nmi/ft/s)	STABLE MEMBER TO SPACECRAFT CONTROL AXES TRANSFORMATION [SC] = [M] [SM]			REFSMMAT [SM] = [REFSMMAT] [BRS] BRS = BASIC REFERENCE SYSTEM			
LOI	ΔV_X	-0.13	-0.70	1	0	0	-0.993	-0.101	-0.066	= Preferred
	ΔV_Y	—	-0.11	0	0.992	-0.126	-0.002	-0.533	0.846	
	ΔV_Z	-0.15	0.11	0	0.126	0.992	-0.120	0.840	0.529	
DOI	ΔV_X	-0.70	—	0.210	-0.012	0.978	0.670	-0.600	-0.437	= Lunar Landing Site
	ΔV_Y	—	—	0.126	0.992	-0.015	0.130	-0.486	0.865	
	ΔV_Z	—	—	-0.970	0.126	0.209	-0.731	-0.636	-0.248	
CSM Circularization	ΔV_X	0.11	0.41	-0.778	-0.037	-0.628	Same as DOI			
	ΔV_Y	—	—	-0.108	0.991	0.075				
	ΔV_Z	0.38	0.16	0.619	0.126	-0.775				
LOPC-1	ΔV_X	—	—	1	0	0	0.193	-0.460	0.867	= Preferred
	ΔV_Y	—	0.73	0	0.992	-0.126	-0.964	0.077	0.255	
	ΔV_Z	—	—	0	0.126	0.992	-0.184	-0.885	-0.429	
TPI	ΔV_X	—	0.72	-0.232	0	-0.973	0.977	-0.040	-0.212	= Lunar Liftoff
	ΔV_Y	—	—	0	1	0	0.167	-0.480	0.861	
	ΔV_Z	—	-0.13	0.973	0	-0.232	-0.136	-0.876	-0.462	
LOPC-2	ΔV_X	—	—	1	0	0	-0.175	0.452	-0.875	Preferred
	ΔV_Y	—	-0.73	0	0.992	-0.126	-0.797	0.457	0.395	
	ΔV_Z	—	-0.07	0	0.126	0.992	0.578	0.766	0.281	
SHAPING	ΔV_X	—	0.53	0.528	0.047	-0.848	Same as TPI			
	ΔV_Y	—	-0.09	-0.089	0.996	0				
	ΔV_Z	0.05	0.53	0.845	0.076	0.530				
TEI	ΔV_X	—	—	1	0	0	0.063	0.756	0.651	= Preferred
	ΔV_Y	—	—	0	0.992	-0.126	0.218	-0.647	0.731	
	ΔV_Z	—	—	0	0.126	0.992	0.974	0.096	-0.206	

-Indicates coefficient is less than 0.05 nmi/ft/s.

ΔV_{SC} ERRORS CAUSED BY GYRO DRIFTS $[\bar{\psi} \times \bar{\lambda}]$ AND PIPA BIASES

GYRO DRIFTS - MERU

PIPA BIASES - CM/SEC^2

ΔV_{SC} - FT/SEC

ΔT_p - SEC (LENGTH OF BURN + 30 SECONDS FOR STARTUP OF AVERAGEG ROUTINE)

ΔT_g - SEC (TIME FROM LAST ALIGNMENT TO BURN)

WEIGHT FACTORS RELATE PIPA BIAS AND GYRO DRIFTS TO VELOCITY ERROR.

LOI

$\Delta V_{x_{SC}} = -3.28 \times 10^{-2}\ AB_x \Delta T_p$

$\Delta V_{y_{SC}} = +2.58 \times 10^{-5}\ NBDY\ \Delta T_g - 2.03 \times 10^{-4}\ NBDZ\ \Delta T_g - 3.25 \times 10^{-2}\ AB_y\ \Delta T_p + 0.41 \times 10^{-2}\ AB_z\ \Delta T_p$

$\Delta V_{z_{SC}} = -2.03 \times 10^{-4}\ NBDY\ \Delta T_g - 2.58 \times 10^{-5}\ NBDZ\ \Delta T_g - .41 \times 10^{-2}\ AB_y\ \Delta T_p - 3.25 \times 10^{-2}\ AB_z\ \Delta T_p$

DOI

$\Delta V_{x_{SC}} = -0.69 \times 10^{-2}\ AB_x\ \Delta T_p - 3.21 \times 10^{-2}\ AB_z\ \Delta T_p$

$\Delta V_{y_{SC}} = -1.46 \times 10^{-5}\ NBDX\ \Delta T_g + 0.19 \times 10^{-5}\ NBDY\ \Delta T_g - 0.32 \times 10^{-5}\ NBDZ\ \Delta T_g - 0.41 \times 10^{-2}\ AB_x\ \Delta T_p - 3.25 \times 10^{-2}\ AB_y\ \Delta T_p + 0.05 \times 10^{-2}\ AB_z\ \Delta T_p$

$\Delta V_{z_{SC}} = -0.19 \times 10^{-5}\ NBDX\ \Delta T_g - 1.49 \times 10^{-5}\ NBDY\ \Delta T_g + 3.18 \times 10^{-2}\ AB_x\ \Delta T_p - 0.41 \times 10^{-2}\ AB_y\ \Delta T_p - 0.69 \times 10^{-2}\ AB_z\ \Delta T_p$

CIRCULARIZATION

$\Delta V_{x_{SC}} = 2.55 \times 10^{-2}\ AB_x\ \Delta T_p + 0.12 \times 10^{-2}\ AB_y\ \Delta T_p + 2.06 \times 10^{-2}\ AB_z\ \Delta T_p$

$\Delta V_{y_{SC}} = +4.50 \times 10^{-6}\ NBDX\ \Delta T_g + 0.92 \times 10^{-6}\ NBDY\ \Delta T_g + 5.63 \times 10^{-6}\ NBDZ\ \Delta T_g + 0.35 \times 10^{-2}\ AB_x\ \Delta T_p - 3.25 \times 10^{-2}\ AB_y\ \Delta T_p - 0.25 \times 10^{-2}\ AB_z\ \Delta T_p$

$\Delta V_{z_{SC}} = +0.78 \times 10^{-6}\ NBDX\ \Delta T_g - 7.20 \times 10^{-6}\ NBDY\ \Delta T_g + 0.55 \times 10^{-6}\ NBDZ\ \Delta T_g - 2.03 \times 10^{-2}\ AB_x\ \Delta T_p - 0.41 \times 10^{-2}\ AB_y\ \Delta T_p + 2.54 \times 10^{-2}\ AB_z\ \Delta T_p$

PLANE CHANGE #1

$\Delta V_{x_{SC}} = -3.28 \times 10^{-2}\ AB_x\ \Delta T_p$

$\Delta V_{y_{SC}} = 0.15 \times 10^{-5}\ NBDY\ \Delta T_g - 1.15 \times 10^{-5}\ NBDZ\ \Delta T_g - 3.25 \times 10^{-2}\ AB_y\ \Delta T_p + 0.41 \times 10^{-2}\ AB_z\ \Delta T_p$

$\Delta V_{z_{SC}} = -1.15 \times 10^{-5}\ NBDY\ \Delta T_g - 0.15 \times 10^{-5}\ NBDZ\ \Delta T_g - 0.41 \times 10^{-2}\ AB_y\ \Delta T_p - 3.25 \times 10^{-2}\ AB_z\ \Delta T_p$

LM TPI

$\Delta V_{x_{SC}} = 0.76 \times 10^{-2}\ AB_x\ \Delta T_p + 3.19 \times 10^{-2}\ AB_z\ \Delta T_p$

$\Delta V_{y_{SC}} = 5.12 \times 10^{-6}\ NBDX\ \Delta T_g + 1.22 \times 10^{-6}\ NBDZ\ \Delta T_g - 3.28 \times 10^{-2}\ AB_y\ \Delta T_p$

$\Delta V_{z_{SC}} = -5.26 \times 10^{-6}\ NBDY\ \Delta T_g - 3.19 \times 10^{-2}\ AB_x\ \Delta T_p + 0.76 \times 10^{-2}\ AB_z\ \Delta T_p$

PLANE CHANGE #2

$\Delta V_{x_{SC}} = -3.28 \times 10^{-2}\ AB_x \Delta T_p$

$\Delta V_{y_{SC}} = 0.26 \times 10^{-5}\ NBDY\ \Delta T_g - 2.04 \times 10^{-5}\ NBDZ\ \Delta T_g - 3.25 \times 10^{-2}\ AB_y \Delta T_p + 0.41 \times 10^{-2}\ AB_z \Delta T_p$

$\Delta V_{z_{SC}} = -2.04 \times 10^{-5}\ NBDY\ \Delta T_g - 0.26 \times 10^{-5}\ NBDZ\ \Delta T_g - 0.41 \times 10^{-2}\ AB_y \Delta T_p - 3.25 \times 10^{-2}\ AB_z \Delta T_p$

SHAPING

$\Delta V_{x_{SC}} = -1.73 \times 10^{-2}\ AB_x \Delta T_p - 0.16 \times 10^{-2}\ AB_y \Delta T_p + 2.78 \times 10^{-2}\ AB_z \Delta T_p$

$\Delta V_{y_{SC}} = 2.34 \times 10^{-6}\ NBDX \Delta T_g + 0.21 \times 10^{-6}\ NBDY\ \Delta T_g - 1.47 \times 10^{-6}\ NBDZ \Delta T_g + 0.29 \times 10^{-2}\ AB_x \Delta T_p - 3.27 \times 10^{-2}\ AB_y \Delta T_p$

$\Delta V_{z_{SC}} = 0.25 \times 10^{-6}\ NBDX \Delta T_g - 2.76 \times 10^{-6}\ NBDY\ \Delta T_g - 2.77 \times 10^{-2}\ AB_x \Delta T_p - 0.25 \times 10^{-2}\ AB_y \Delta T_p - 1.74 \times 10^{-2}\ AB_z \Delta T_p$

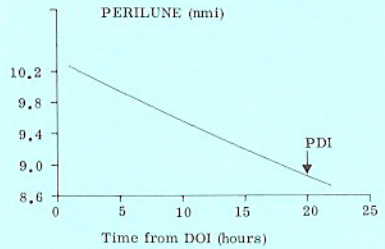
TEI

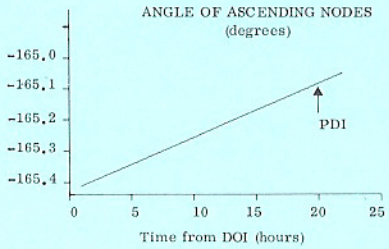
$\Delta V_{x_{SC}} = -3.28 \times 10^{-2}\ AB_x \Delta T_p$

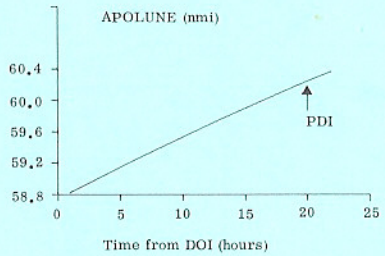
$\Delta V_{y_{SC}} = -0.30 \times 10^{-4}\ NBDY\ \Delta T_g + 2.32 \times 10^{-4}\ NBDZ\ \Delta T_g + 3.25 \times 10^{-2}\ AB_y\ \Delta T_p - 0.41 \times 10^{-2}\ AB_z\ \Delta T_p$

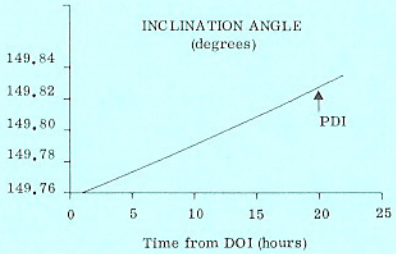
$\Delta V_{z_{SC}} = +2.32 \times 10^{-4}\ NBDY\ \Delta T_g + 0.30 \times 10^{-4}\ NBDZ\ \Delta T_g + 0.41 \times 10^{-2}\ AB_y\ \Delta T_p + 3.25 \times 10^{-2}\ AB_z\ \Delta T_p$

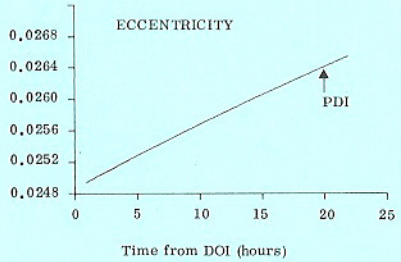
APOLLO 16 LUNAR ORBIT PARAMETERS (TYPICAL)

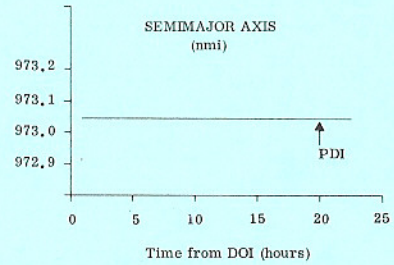












EFFECT OF INSTRUMENT ERRORS ON
LUNAR LANDING POSITION
APOLLO 16

INSTRUMENT ERROR	CROSS-RANGE ERROR (ft)	DOWN RANGE ERROR (ft)
PIPA BIAS (cm/sec^2)		
X	0	-178
Y	-2985	0
Z	0	-2984
IMU MISALIGNMENTS (mrad.)		
About X_{SM}	777	0
About Y_{SM}	0	-86
About Z_{SM}	42	0
GYRO DRIFT (meru)*		
NBDX	10.4	0
NBDY	0	-3.2
NBDZ	2.8	0

* Gyro drift was assumed to start at ignition.

Effect of Instrument Errors on LM Insertion Orbit

Apollo 16

Instrument Error	Radial Velocity Error (ft/s)	*Crossrange Velocity Error (ft/s)	Downrange Velocity Error (ft/s)	Apolune Error (ft)	Perilune Error (ft)	Crossrange Error (ft)	Inclination Error (deg)
PIPA Bias (cm/s^2)							
ACBX	-14.0	–	2.4	Graph A	Graph A	–	–
ACBY	–	-13.5	–	–	–	-2,947	-0.097
ACBZ	-5.3	–	-13.4	Graph B	Graph B	–	–
IMU Misalignment at Liftoff (arc-seconds)							
ϕ_X	–	-0.026	–	–	–	-4.8	-1.8×10^{-4}
ϕ_Y	0.024	–	-0.011	Graph C	Graph C	–	–
ϕ_Z	–	0.006	–	–	–	2.2	5.0×10^{-4}
Gyro Drift Starting at Liftoff (meru)							
NBDX	–	-0.099	–	–	–	-12.4	-6.3×10^{-4}
NBDY	0.094	–	-0.025	Graph D	Graph D	–	–
NBDZ (+ @ $-Z_{SM}$)	–	-0.008	–	–	–	-3.2	-6.7×10^{-5}

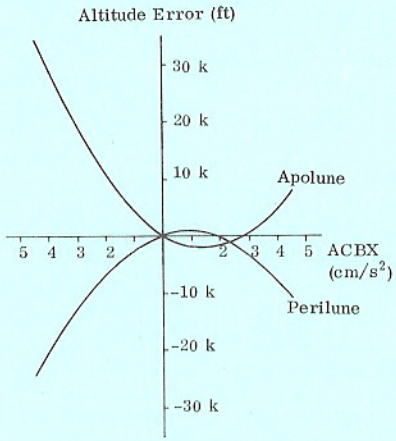
*$\overline{CR} = \overline{DR} \times \overline{R}$

NOMINAL VALUES:

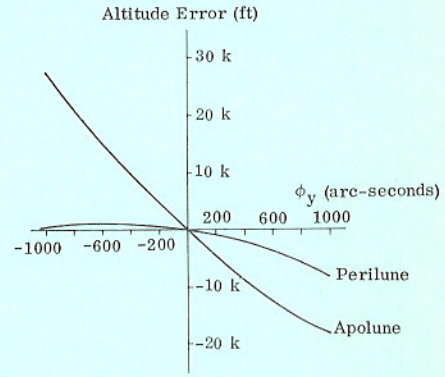
Radial Velocity	32 ft/s	Apolune	45.6 nmi
Crossrange Velocity	0 ft/s	Perilune	9.0 nmi
Downrange Velocity	5,531.3 ft/s	Crossrange	0 nmi
		Inclination	147 deg

LM INSERTION ORBIT APOLUNE AND PERILUNE ERRORS

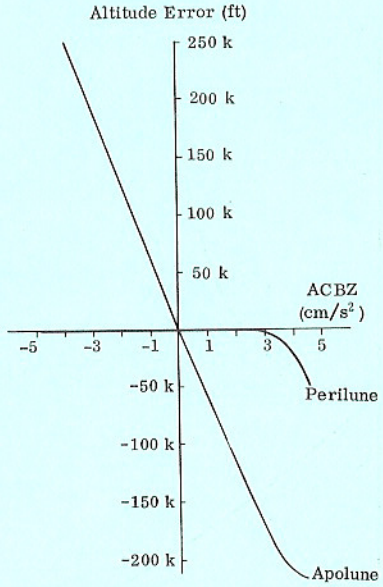
APOLLO 16



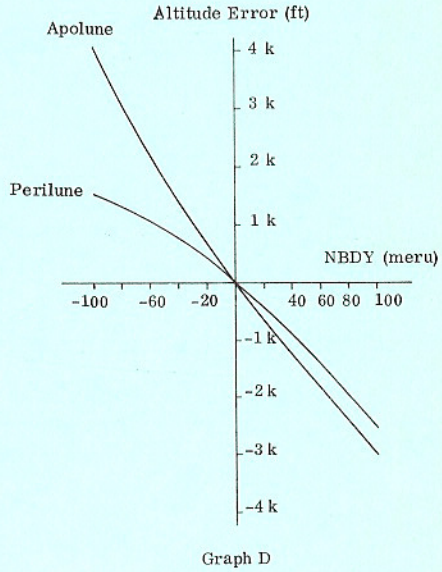
Graph A



Graph C

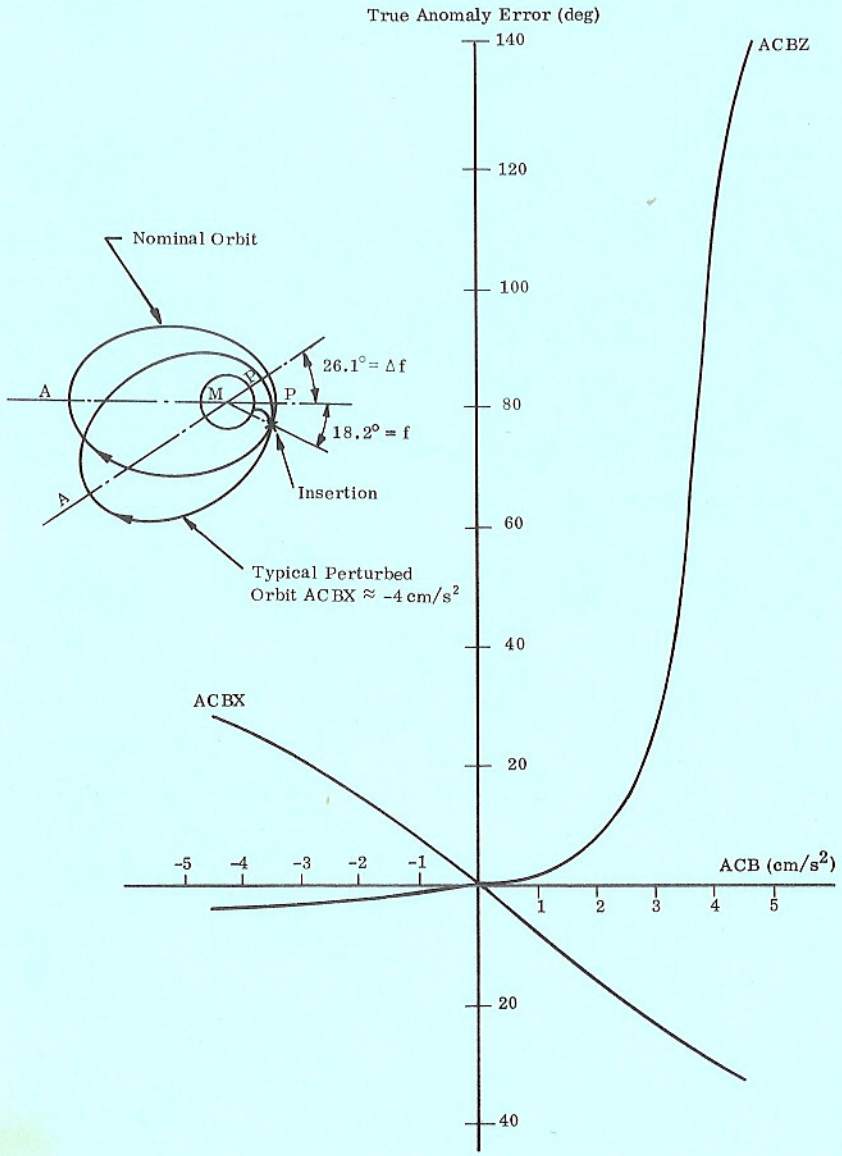


Graph B



Graph D

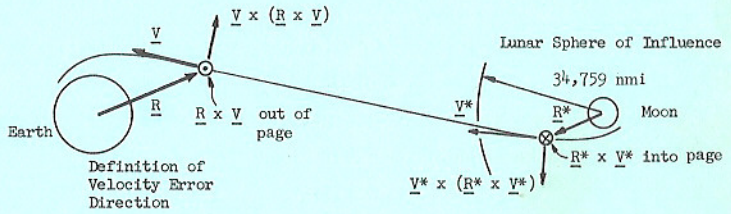
LM INSERTION ORBIT TRUE ANOMALY ERRORS DUE TO
UNCOMPENSATED PIPA BIAS
APOLLO 16



ERRORS AT THE ENTRY INTERFACE (400,000 FT. ALT.)
DUE TO VELOCITY ERRORS DURING TRANSEARTH COAST

DIRECTION OF VELOCITY ERROR	ERROR IN VACUUM PERIGEE (nmi/ft/s)	ERROR IN TIME OF ARRIVAL (s/ft/s)	ERROR IN VELOCITY AT EI (ft/s/ft/s)	ERROR IN FLIGHT PATH ANGLE AT EI (degrees/ft/s)
ERROR EXISTING AT TEI CUTOFF (222:23:03.3 GET)				
Along $\underline{V}^*$	-22.89	-96.95	2.84	-1.95
Along $\underline{R}^* \times \underline{V}^*$	2.04	10.00	-0.05	0.14
Along $\underline{V}^* \times (\underline{R}^* \times \underline{V}^*)$	-7.02	0.35	0.05	-0.49
ERROR EXISTING AT MCC-5 (TEI + 17 hours) (239:23:03.3 GET)				
Along $\underline{V}$	1.67	-21.60	0.05	0.12
Along $\underline{R} \times \underline{V}$	0.68	1.85	0.02	0.05
Along $\underline{V} \times (\underline{R} \times \underline{V})$	9.97	30.90	-0.04	0.72
ERROR EXISTING AT MCC-6 (EI - 22 hours) (268:22:45.4 GET)				
Along $\underline{V}$	1.24	-5.25	0.23	0.09
Along $\underline{R} \times \underline{V}$	0.02	0.05	0.02	0
Along $\underline{V} \times (\underline{R} \times \underline{V})$	5.93	14.85	0.01	0.41
ERROR EXISTING AT MCC-7 (EI - 3 hours) (287:22:45.4 GET)				
Along $\underline{V}$	0.50	0.30	0.51	0.04
Along $\underline{R} \times \underline{V}$	0	0	0	0
Along $\underline{V} \times (\underline{R} \times \underline{V})$	1.48	2.55	0.01	0.10

* $\underline{R}$ and $\underline{V}$ are with respect to moon centered inertial system



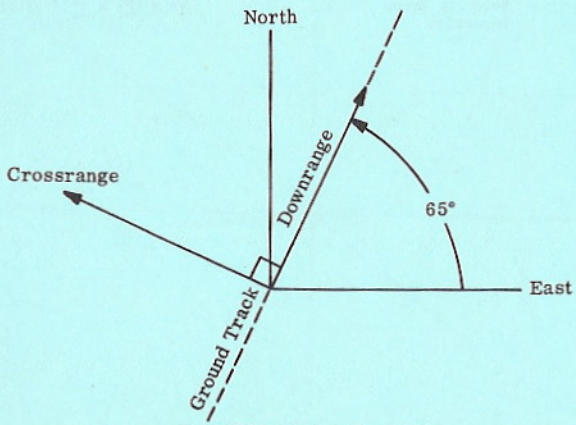
TYPICAL LUNAR RETURN ENTRY POSITION ERROR SENSITIVITIES AT GUIDANCE TERMINATION

ERROR SOURCE (ϵ)		NOMINAL 1,250 NMI ENTRY	
		Downrange Error (nmi/ϵ)	Crossrange Error (nmi/ϵ)
GYRO BIAS DRIFT (meru)	NBDX	0.0	-0.17
	NBDY	-0.12	0.0
	NBDZ	0.0	-0.43
ACCELEROMETER BIAS (cm/sec^2)	ACBX	-4.49	0.0
	ACBY	0.0	4.65
	ACBZ	-1.45	0.0

NOTES:
1. Assumed accelerometer operation 15 minutes prior to EI (altitude of 400,000 feet).
2. Assumed IMU drift from 1.5 hours prior to EI.
3. Assumed IMU orientation such that at EI:

$$\bar{Z}_{SM} = -\text{Unit}(\bar{R})$$
$$\bar{Y}_{SM} = -\text{Unit}(\bar{R} \times \bar{V})$$
$$\bar{X}_{SM} = \bar{Y}_{SM} \times \bar{Z}_{SM}$$

4. Crossrange-downrange definition:



Nominal 1,250 Nautical Miles Entry