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DIGITALLY CONTROLLED PULSE TORQUING OF PRECISION INERTIAL INSTRUMENTS

by

Frank E. Gauntt

September 1957

INSTRUMENTATION
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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Cambridge 39, Mass.

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TABLE OF CONTENTS

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DIGITALLY CONTROLLED PULSE TORQUING OF PRECISION INERTIAL INSTRUMENTS

INTRODUCTION

A test was performed to determine the feasibility of pulse torquing. A small special purpose digital computer was coupled to the torque motor of a precise inertial gyro by a power switch in place of digital-to-analog conversion equipment. The idea was to control the average current to the torque motor by holding its magnitude constant and reversing the direction of current flow in some prescribed cyclic manner. A brief sketch comparing a pulse torque system to an analog torque system is given in Appendix A.

PROCEDURE

The test consisted of measuring the torque applied about the output axis of the gyro as the input to the digital computer was varied. The gyro used was a 45 FG 4 on an "input axis horizontal" servo run.(1) The applied torque was determined by measuring the turntable rate and computing out the effects of earth rate and gyro mass unbalance.

The input to the computer was the number stored in its memory and will be referred to as the control number, y. The special purpose digital computer used in the test was a D.D.A. (Digital Differential Analyzer; Refer to Appendix C) and its memory was its Y register.

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DATA REDUCTION

The relationship between the applied torque, T(y), and the control number, y, was assumed to be linear and is stated in the form:

$$T(y) \quad = Ky + B \qquad \text{(Refer to Appendix B)}$$

$$+1 \geq y \geq -1$$

$$K \gg B$$

The constants K and B were determined from the torque for y = +1 and y = -1 which were measured at the beginning of the test.

$$K \quad = \frac{T(+1) - T(-1)}{2}$$

$$B \quad = \frac{T(+1) + T(-1)}{2}$$

From the above expressions, and the control numbers used in the performance of the test, expected values of torque were computed and are referred to as the predicted torque.

The measured torque as a function of the control number was compared to the corresponding values of predicted torque. The error in the measured torque from the predicted torque was defined as:

$$\text{Error as percentage of Full Torque} = \frac{T\,\text{meas} - T\,\text{pred}}{K} \times 100$$

and was plotted as a function of time.

[redacted]

RESULTS TEST #1

The test was first run with a basic clock interval of 1000 microseconds. This is the shortest possible interval between successive reversals of current direction for a given computer speed. A plot of the error in the measured torque from the predicted torque, assuming a linear relation between torque and the control number, is shown in Figure 1. The drift based on the full torque of 4500 dyne-cm. was +0.06 % for the first 28 hours and +0.2 % for the total of 48 hours.

These results were accomplished with a power switch that was designed for no better than 1% drift. A more thorough design will be undertaken to improve the current stability. Consideration will also be given to possible techniques for reduction of the restriction on current stability.(2)

RESULTS TEST #2

The test was repeated using a basic clock interval of 150 microseconds. The characteristic time of the torque motor was 50 microseconds. Operation was then within the switching transient; therefore, it was expected that the results would not be as good as that of Test #1. Photographs of current waveform for "zero" torque are shown for both tests in Figure 2.

The results of this test are shown in Figure 3. These results indicate that a momentary failure in the test system occurred within the interval from 36 to 50 hours. A power switch failure would be either partial or total, the results of which would be quite different from that observed in the test. Therefore, the results observed in this interval are not indicative of the performance of the power switch and should be considered invalid. An error of less than + 2 % of full torque of 4900 dyne-cm. may be taken as representative of the power switch performance for the basic clock interval of 150 microseconds. This is about ten times larger than that for the test at 1000 microseconds, a result that was not unexpected.

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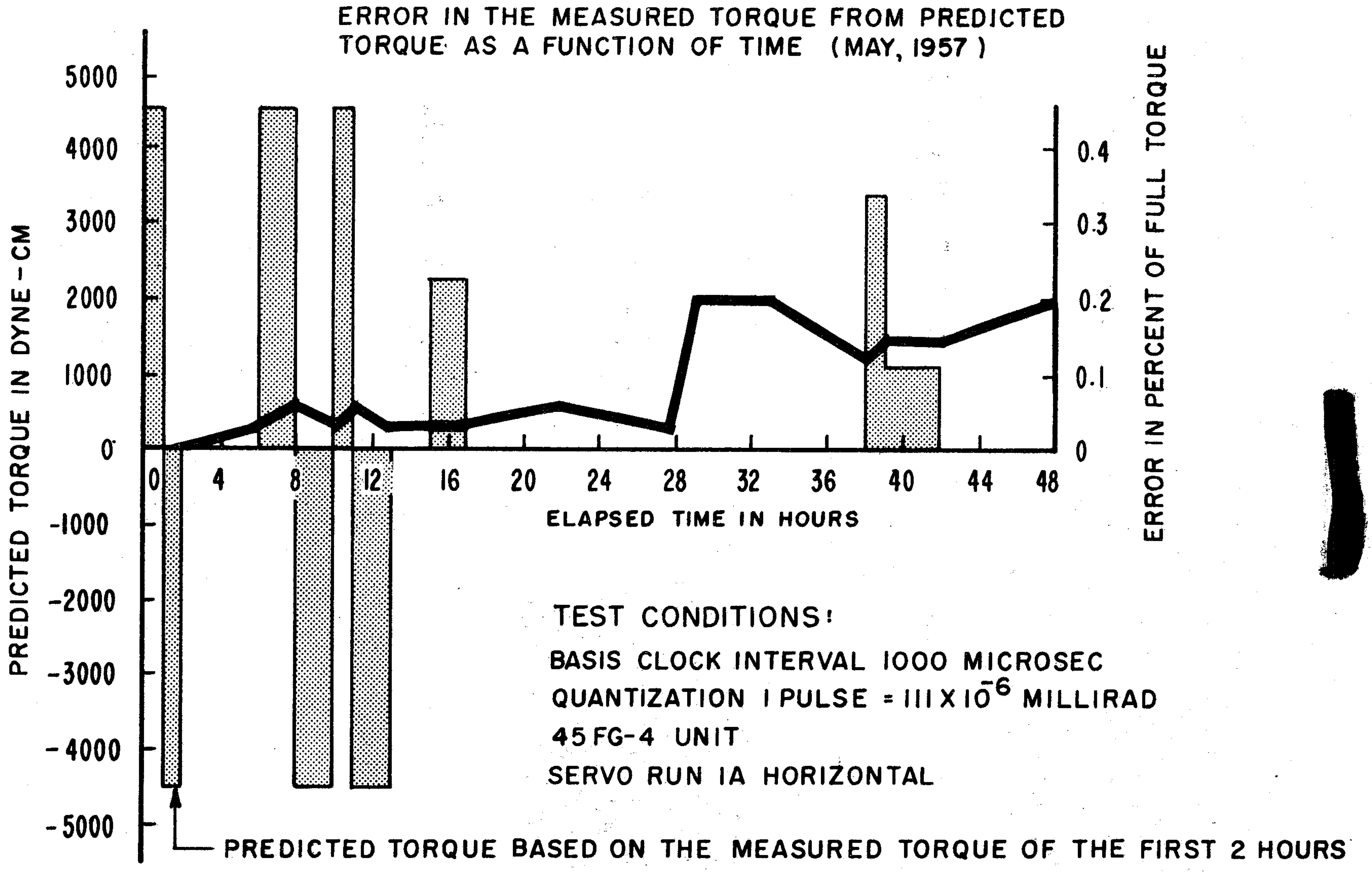

Fig. 1 Results of Pulse Torquing an Inertial Gyro

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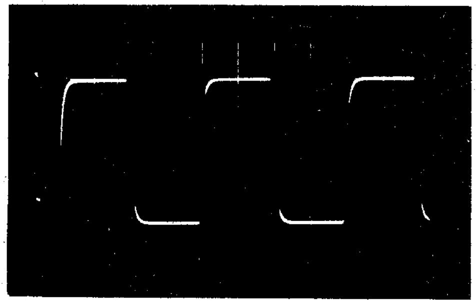
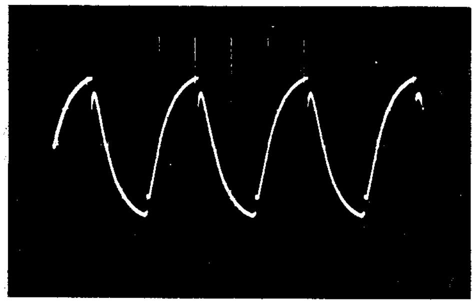


2 VOLTS /cm

500 MICROSEC /cm

BASIC CLOCK INTERVAL 1000 MICROSEC



2 VOLTS /cm

100 MICROSEC /cm

BASIC CLOCK INTERVAL 150 MICROSEC

Fig. 2 Waveform of Current in Switching Winding for "Zero" Torque

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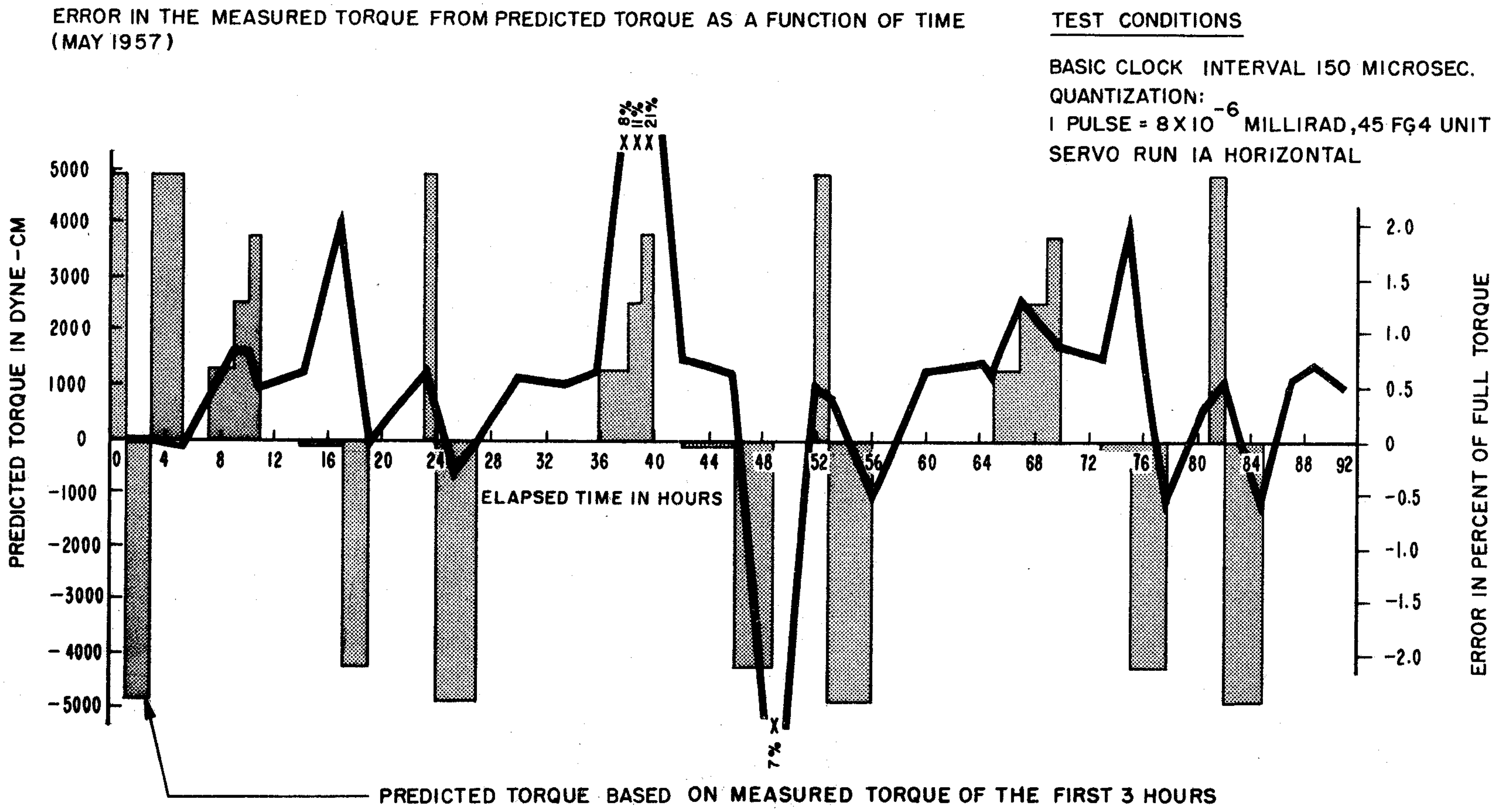

Fig. 3 Results of Pulse Torquing an Inertial Gyro

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APPENDIX A

A hypothetical example showing the relation between pulse torquing and analog torquing is displayed in Figure 4. From the gyroscopic equation, the relation between the torque applied about an axis orthogonal to the spin reference axis, $T(t)$, and the precessional angular velocity, $\omega(t)$, about the axis orthogonal to the torquing axis and the spin reference axis may be shown as:

$$\omega(t) = \frac{d\,\theta(t)}{dt} = S\,T(t)$$

$\theta(t)$ = angle of precession in time t due to torque $T(t)$

S = $(\text{angular momentum of gyro wheel})^{-1}$

= 1/8 milliradian per millisec per dyne-cm

In order to generate the function $\theta(t)$, the solid curve of Figure 4a, the analog torque function $T(t)$, Figure 4b, must be applied about the torquing axis.

If the broken line curve of Figure 4a is an acceptable approximation to the function $\theta(t)$, differing from it by no more than $\pm$ 0.5 milliradians, the pulse torque pattern of Figure 4c may be applied about the torquing axis in place of the analog torque function. If the sensitivity of the torque motor is chosen as 1 dyne-cm per milliamp, Figures 4b and 4c may also represent the waveform of the current to the torque motor.

It may be noted that the average value of the pulse pattern

[redacted]

[redacted]

over any 8 millisecond interval, in which the corresponding analog function is constant, is equal to the value of the analog function.

It may also be noted that, since the heat dissipation in the torque motor is proportional to the square of the current, the dissipation in the pulse system is constant while in the analog system it is proportional to the square of the analog function.

The maximum deviation of ± 0.5 milliradians is referred to as the quantization of the approximation and is equal to the angle, $\Delta\theta$, generated by one pulse of a minimum width of one millisecond. This minimum pulse width is referred to in the body of the report as the basic clock interval.

If it is desired to improve the approximation, it is only necessary to reduce the quantization. This is done by reducing the basic clock interval. In a physical system, since the torque motor is not perfectly linear, it is necessary to limit the minimum basic clock interval to at least ten torque motor time constants for reliable performance. As long as this restriction is met, the nonlinearity of the torque motor is of no appreciable concern. (Refer to Appendix B.)

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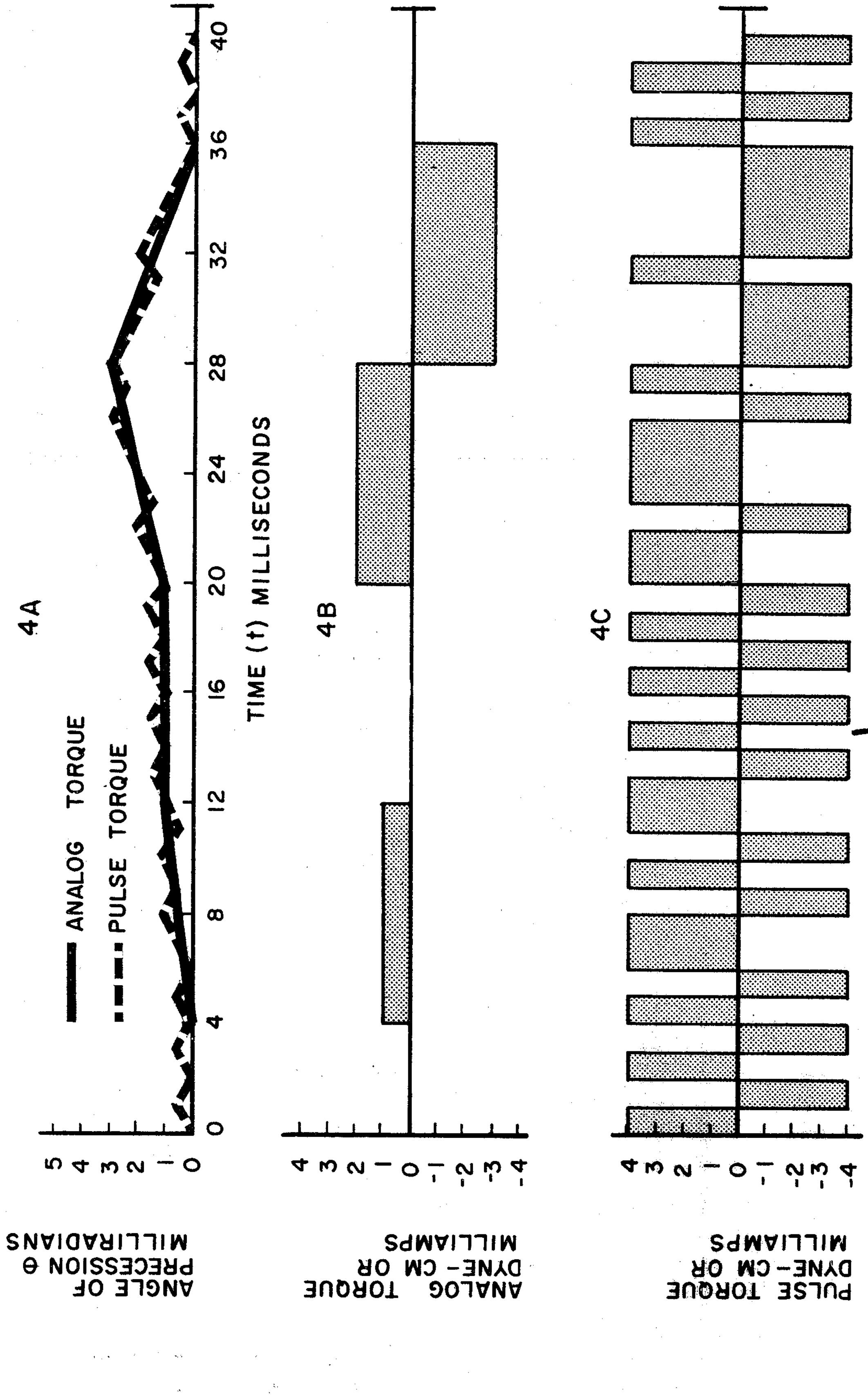

Fig. 4 A Comparison of Pulse Torquing and Analog Torquing

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APPENDIX B

(Refer to Figures 4c and 5)

The average torque T, over any interval N, may be found from the relation:

$$T = K_1 \frac{n_1}{N} - K_2 \frac{n_2}{N} \qquad (1)$$

$$N = n_1 + n_2$$

n_1 = number of basic clock intervals that torque motor current flow is positive

n_2 = number of basic clock intervals that torque motor current flow is negative

$$K_1 = S_{TG} I_R (I_S + \Delta I_S) \qquad I_S \gg \Delta I_S$$

$$K_2 = S_{TG} I_R (I_S - \Delta I_S)$$

S_{TG} = Torque Motor Sensitivity

I_R = Magnitude of current in reference winding

$$I_S = \frac{I_{S+} + I_{S-}}{2}$$

$$\Delta I_S = \frac{I_{S+} - I_{S-}}{2}$$

I_{S+} = Magnitude of current in switching winding for positive current flow

[redacted]

I_{S-} = Magnitude of current in switching winding for negative current flow

Equation (1) may be rewritten:

$$T = K \frac{n_1 - n_2}{N} + B \qquad (2)$$

where

$$K = \frac{K_1 + K_2}{2} = S_{TG} I_R I_S$$

$$B = \frac{K_1 - K_2}{2} = S_{TG} I_R \Delta I_S$$

Assuming a linear torque motor, the only effect of unbalanced switching current is the additive constant B. The existence of this constant does not destroy the linearity of the pulse torquing system.

Let the control number y, as previously defined, be expressed as:

$$y = \frac{n_1 - n_2}{N}$$

The final form of the torque-control number equation is then:

$$T(y) = Ky + B$$

The significance of a nonlinear torque motor is that its sensitivity, S_{TG}, is not necessarily the same for both positive and negative switching current. If the following definitions are made:

S_{TG+} = Torque Motor sensitivity for positive switching current

[redacted]

S_{TG-} = Torque Motor sensitivity for negative switching current

$$S_{TG} = \frac{S_{TG+} + S_{TG-}}{2}$$

$$\Delta S_{TG} = \frac{S_{TG+} - S_{TG-}}{2}$$

and the outline of the preceeding discussion is reviewed, it is found that :

$$K = S_{TG} I_R I_S + I_R \Delta S_{TG} \Delta I_S$$

$$B = S_{TG} I_R \Delta I_S + I_R I_S \Delta S_{TG}$$

K and B are still constants. The only effect of a nonlinear torque motor on the equation (3) is a change in the constants K and B. This does not introduce any nonlinearities into the torque-control number relationship.

If ΔS_{TG} is not too large, the constant B may be reduced to zero by unbalancing the switching current so that:

$$\frac{\Delta I_S}{I_S} = - \frac{\Delta S_{TG}}{S_{TG}}$$

Then:

$$K = S_{TG} I_R I_S \left[1 - \left(\frac{\Delta I_S}{I_S} \right)^2 \right]$$

Another method of effectively reducing B to zero is to measure it accurately and compensate for it in the program of the digital computer.

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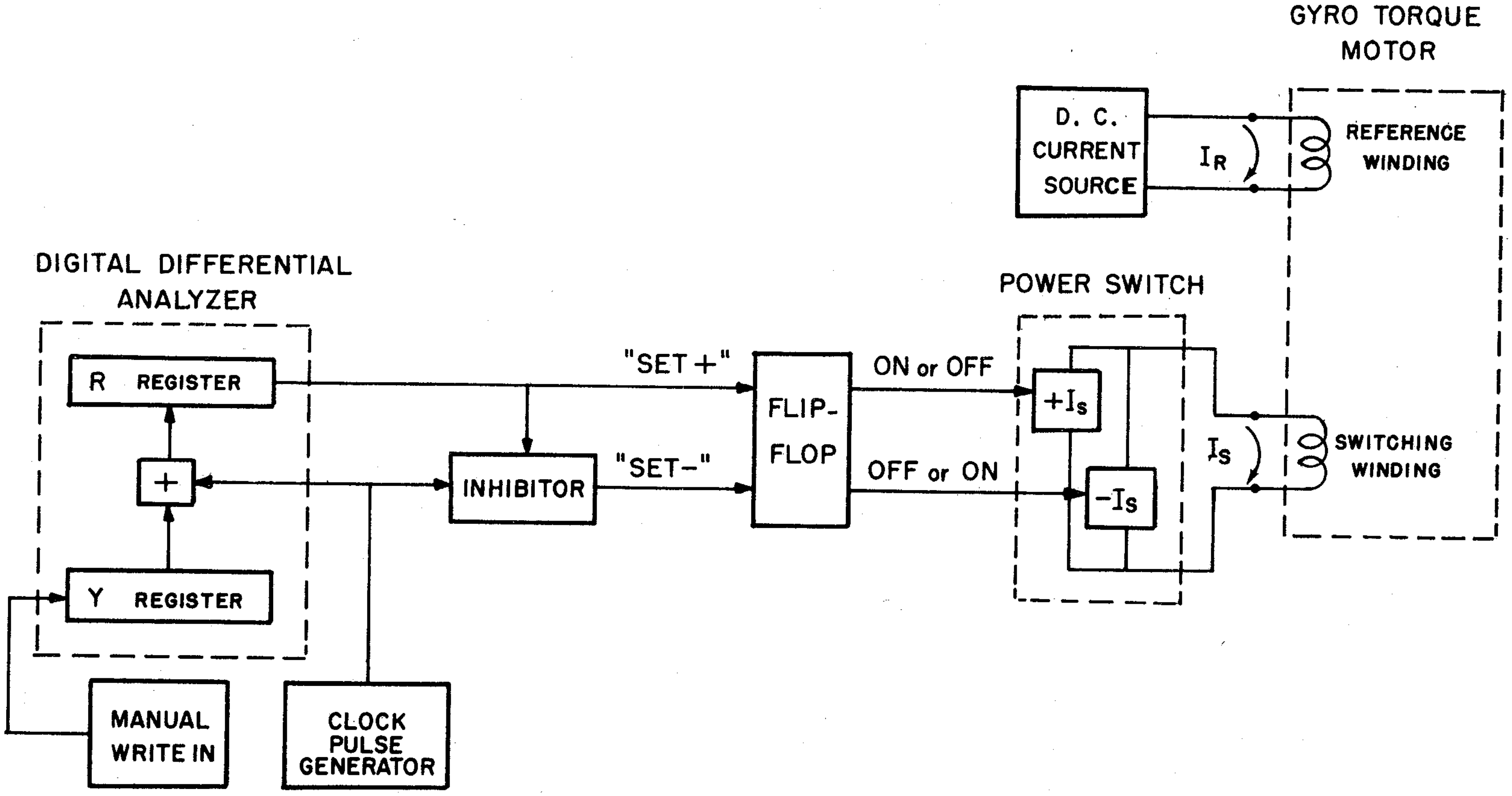

Fig. 5 Block Diagram - Digitally Controlled Pulse Torquing System

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APPENDIX C

A block diagram of the pulse torquing system used for this test is shown in Figure 5. The direction of current flow in the switching winding of the torque motor is determined by the state of the flip-flop. The pulses on the "set +" line are the overflow pulses from the R register of the D.D.A.(3) and when they occur are coincident with the pulses from the clock pulse generator. The period of the clock pulse generator is the basic clock interval.

When a pulse appears on "set +", the inhibitor (a normally open gate) blocks the clock pulse to the "set -" line, thereby setting the flip-flop to the "+" state. This causes the switching current to flow in the positive direction. If on a clock pulse, the R register does not overflow, the clock pulse passes through the inhibitor to the "set -" line and sets the flip-flop to the " - " state which, in turn, causes the switching current to flow in the negative direction.

An overflow occurs whenever the results of an addition to the contents of the R register exceed its capacity. On a clock pulse, the contents of the Y register are added to the contents of the R register. In this manner, the rate of overflow pulses is controlled by the magnitude of the number stored in the Y register. The capacity of the Y register is, normally, equal to that of the R register.

The range of the control number y, -1 to +1, is scaled to fill the Y register. For a desired amount of torque, the corresponding control number is scaled and stored in the Y register. The number of overflow pulses in the interval N is equal to the number

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[redacted]

n_1 as defined in Appendix B. The relationship between the control number and the number of overflow pulses is :

$$n_1 = \frac{y+1}{2} N$$

[redacted]

[redacted]

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